Dualization and Discretization of Linear-Quadratic Control Problems with Bang-Bang Solutions

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Abstract: We consider linear-quadratic (LQ) control problems, where the control variable appears linearly and is box-constrained. It is well-known that these problems exhibit bang-bang and singular solutions. We assume that the solution is of bang-bang type, which is computationally challenging to obtain. We employ a quadratic regularization of the LQ control problem by embedding the L^2-norm of the control variable into the cost functional. First, we find a dual problem guided by the methodology of Fenchel duality. Then we prove strong duality with zero duality gap and a saddle point property, which together ensure that the primal solution can be recovered from the dual solution. We propose a discretization scheme for the dual problem, under which a diagram, depicting the relations between the primal and dual problems and their discretization, commutes. The commuting diagram ensures that, given convergence results for the discrete primal variables, the discrete dual variables also converge to a solution of the dual problem with a similar error bound. Numerical experiments conclude the talk. We demonstrate that significant computational savings can be achieved by solving the dual, rather than the primal, problem.

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