A Distributed Jacobi Algorithm for Optimization Problems with Coupled Constraints

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Abstract: We consider a large-scale convex optimization problem that involves a group of agents, under general affine constraints:

$$\begin{aligned} \min_{x \in X} \quad & \textstyle\sum_{i=1}^{M} f_i(x) \\ \text{s.t.} \quad & h_k(x) = 0, \quad k = 1, \cdots, N_{\text{eq}} \\ & g_k(x) \leq 0, \quad k = 1, \cdots, N_{\text{ineq}} \end{aligned}$$

with $x = [x_1, \cdots, x_M]^T$, $X = X_1 \times \cdots \times X_M$, and $X \subset \mathbb{R}^n$.

This problem often arises in distributed model predictive control applications. We propose a distributed Jacobi algorithm to solve this problem in a cooperative manner. In every iteration, each agent solves its local problem and exchanges information with its 'direct neighbours'. After that, the new and the old solutions are used in a convex combination to maintain feasibility at every iteration. The convex combination update is equivalent to a scheme using only local updates. Therefore, the algorithm only needs neighbour-to-neighbour communications and local computations, making it suitable for distributed implementation.

We provide the *a posteriori* certification for centralized optimality of distributed solutions, based on comparing Lagrange multipliers of the local problems. Furthermore, we also prove *a priori* conditions that guarantee convergence to optimality in several problem settings.

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