Computing Statistical Solutions of Hyperbolic Conservation Laws

U. Fjordholm[1], K. Lye[2], and S. Mishra[2]

Abstract: An open question in the field of hyperbolic conservation laws is the question of well- posedness. Recent theoretical and numerical evidence have indicated that the correct notion of solutions for systems of conservation laws is the notion of measure valued solutions. These have later been extended to include statistical solutions, which incorporates the measure valued solutions along with multi-point spatial correlations measures.

We review the theory of statistical solutions for conservation laws. Afterwards, we introduce a convergent numerical method for computing the statistical solution of conservation laws, and prove that it converges in the Wasserstein distance through narrow convergence for the case of scalar conservation laws.

For the scalar case, we validate our theory by computing the structure functions of the Burgers equation with random initial data. We especially focus on Brownian initial data, and the measurement of the scalings of the structure functions. The results agree well with the theory, and we get the expected convergence rate. We furthermore show that we can get faster computations using Multilevel Monte-Carlo for computing the statistical solutions of scalar conservation laws.

In the case of systems of equations, we test our theory against the compressible Euler equations in two space dimensions. We check our numerical algorithm against two ill-behaved initial data, the Kelvin-Helmholtz instability and the Richtmeyer-Meshkov instability, and compute the corresponding structure functions.

[1] Department of Mathematics, University of Oslo,
Postboks 1053 Blindern,
0316 Oslo, Norway
ulriksf@math.uio.no

[2] Seminar for Applied Mathematics, ETH Zürich,
Rämistrasse 101,
8092 Zürich, Switzerland
{*kjetil.lye, siddhartha.mishra*}*@sam.math.ethz.ch*