# Effective models for reaction-diffusion processes including membrane transport and metabolic channeling

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**Abstract:**

In this talk, we derive a homogenized model for the carbohydrate metabolism in plant cells. We start from a microscopic model on the subcellular scale, which takes into account the complex geometry and the micro-physical processes inside the cell. Of special interest in this metabolic pathway are the chloroplast and the mitochondria, which are distributed in a high number within the cytosol in the cell. The ratio between the length of these organelles and the whole cell is given by the parameter $\epsilon$. The processes in the different compartments are described by a system of reaction-diffusion equations and the exchange of chemical substances between the organelles and the cytosol are given by nonlinear transmission conditions across the membrane of the chloroplasts and the mitochondria, respectively. Additionally, we take into account the effect of metabolic channeling, which takes place at the outer membrane of the mitochondria. In our model, this phenomenon is described by a Wentzell-boundary condition at the mitochondrial membrane. Our aim is to derive rigorously a macroscopic model on the cellular scale, the solution of which approximates the solution of the microscopic model.

From a mathematical point of view, we consider a periodic multi-component porous medium $\Omega$, the whole cell, where the components are separated by an interface $\Gamma_\epsilon$. One component $\Omega_1^\epsilon$ is connected, the other one $\Omega_2^\epsilon$ is disconnected and consist of periodically distributed inclusions, the chloroplasts and the mitochondria. The parameter $\epsilon$ describes the periodicity of the distribution and the ratio between the length of the inclusions and the whole domain $\Omega$. The partial differential equations in the different domains are coupled by nonlinear transmission conditions of Neumann- or Wentzell-type. In the latter case, the normal fluxes at the interface are given by a reaction–diffusion equation on the interface $\Gamma_\epsilon$, with nonlinearities depending on the solutions on both sides of the interface. For $\epsilon \to 0$, we derive a macroscopic model, where the crucial point is to pass to the limit in the nonlinear boundary terms.

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