A Local Mesh Modification Strategy for Interface Problems with Application to Shape and Topology Optimization

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Abstract: In topology and shape optimization one is often interested in finding the optimal layout of a subdomain of a fixed computational domain. Most algorithms start out from an initial design and use sensitivity information of the objective functional with respect to the geometry in order to successively update the material interface to reach an optimum. Examples for such sensitivities are the topological derivative or the shape derivative. In PDE-constrained design optimization problems, these sensitivities usually depend on the solutions to the state equation and the adjoint equation, which have to be determined in each iteration of the optimization algorithm. When these two partial differential equations are solved by a standard finite element method, it is important that the material interface is resolved by the finite element mesh in order to obtain accurate approximate solutions.

We present a local mesh modification strategy which adapts mesh nodes only in a vicinity of the material interface in such a way that, on the one hand, the interface is resolved accurately, and on the other hand, no angle can come too close to 180 degrees. This maximum angle condition allows to show optimal order of convergence of the finite element method independently of the location of the interface relative to the mesh. While the occurrence of too large angles is excluded by the procedure, angles can become arbitrarily small which affects the condition of the arising linear systems. This approach is an adaptation of the approach of Frei and Richter, where finite element methods on quadrilateral meshes are used, to the case of piecewise linear, globally continuous finite elements on triangular grids.

We integrate this interface finite element method into a two-stage design optimization algorithm where the optimal topology is found by means of a level set algorithm that is based on the topological derivative before using shape optimization as a postprocessing. We apply the presented optimization strategy to the design optimization of an electric motor.

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