New Class of Mathematical Models for Biomass Development and Processing

M. Gokieli[1], N. Kenmochi[2], and M. Niezgodka[3]

Abstract: Our interest is mathematical modeling of nature's mechanisms, especially with respect to "biomass", which is said to be one of potentially unlimited energy resources for the next generation. We intend to model biomass from its growth process to utilization, considering it in its environment — fluid, which can be gas or liquid. We want to model coupled processes of fluid dynamics, transport and reactions of diverse biological and chemical components inside, aggregation, biomass growth, and thermal processes.

We consider in particular two case studies: biomass growth and reproduction on one hand, and biomass conversion via thermal processes of pyrolysis, gasification and combustion on the other. These two case studies refer to different situations, but their models have lot in common. An essential feature of the modelled flows is that they are multi–phase (phases can be e.g. biomass, water or gas) and that the boundaries between phases evolve. This also holds for the flow's domain boundaries. The flow's domain may locally narrow as a result of biomass aggregating on its walls, or broaden out by biomass combustion. Furthermore, these free boundaries are exactly the place where crucial chemical reaction occur. This is an important challenge for the formulation of these models as well as for their analysis,

In order to cope with these difficulties we replace sharp interfaces by thick layers, considered as mixture of phases, and a function which will have the role of *constraint*. A first approach to the construction of such a model and the analysis of its well–posedeness can be found in our recent work [1,2].

We believe that the idea of introducing a thick layer as a mixture of solid and liquid and treating the free boundaries as convex constraints, makes a breakthrough in the analysis of free boundary problems in fluid flows and systems of coupled equations including fluid flows. We expect it to allow us to treat our problem rigorously, in a form of variational inequality of Navier-Stokes type. It should be noted that our model is acceptable from the physical and biological point of view.

References

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[1,2,3] Interdisciplinary Centre for Mathematical and Computational Modelling, University of Warsaw
Tyniecka 15 / 17, 02-630 Warsaw, Poland
m.gokieli@icm.edu.pl, *n.kenmochi@icm.edu.pl*, *m.niezgodka@icm.edu.pl*