

# Numerical Optimal Control of Time-Varying Dynamical Systems with Applications in Building Operation

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**Abstract:** Residential and office buildings take up a large fraction of our energy consumption. It is interesting to reduce the amount of energy used in the operation of a building both from an ecological and economic point of view. Modeling and optimization can help identify where the greatest potential for significant energy savings lies.

We present models of different accuracy that depict reality with varying degrees of realism. These models can range from simple one dimensional models describing the mean temperature in a single room on a very coarse and abstract level to large-scale, highly detailed PDE simulations of the airflow and temperature within a building complex. By taking into account constantly changing data such as the outside temperature the model becomes time-varying which makes the problem more challenging.

Our goal is to develop control strategies that keep the temperature within the building inside a given range and at the same time minimize the energy for heating or cooling, i.e. to achieve the goal in an energy-efficient manner. We are particularly interested in the long-term performance of the method.

We solve the problem using economic model predictive control (MPC), which is a more general control concept than classical stabilizing MPC, and which directly incorporates an economic criterion in the cost function and also enables us to deal with the time-varying dynamics in a natural way.

From the theoretical point of view, the turnpike property plays an important role in proving long-term performance estimates. We will address this in the talk by showing simulations that demonstrate that the turnpike property can be observed numerically.

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