

# Second-Order Variational Analysis in Second-Order Cone Programming

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**Abstract:** The paper conducts a second-order variational analysis for an important class of nonpolyhedral conic programs generated by the so-called second-order/Lorentz/ice-cream cone  $\mathbb{Q}$ . From one hand, we prove that the indicator function of  $\mathbb{Q}$  is always twice epi-differentiable and apply this result to characterizing the uniqueness of Lagrange multipliers at stationary points together with an error bound estimate in the general second-order cone setting involving  $\mathcal{C}^2$ -smooth data. On the other hand, we precisely calculate the graphical derivative of the normal cone mapping to  $\mathbb{Q}$  under the weakest metric subregularity constraint qualification and then give an application of the latter result to a complete characterization of isolated calmness for perturbed variational systems associated with second-order cone programs. The obtained results seem to be the first in the literature in these directions for nonpolyhedral problems without imposing any nondegeneracy assumptions.

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