Accurate Front Capturing Schemes for Tumor Growth Models Equation with a Free Boundary Limit

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Abstract: We consider a class of tumor growth models under the combined effects of density-dependent pressure and cell multiplication. When the pressure-density relationship becomes highly nonlinear, it converges to a free boundary model as a singular limit. In particular, the constitutive law connecting pressure p and density ρ is $p(\rho) = \frac{m}{m-1}\rho^{m-1}$, when $m \gg 1$, the cell density ρ may evolve its support due to a pressure-driven geometric motion with sharp interface along the boundary of its support. The nonlinearity and degeneracy in the diffusion bring great challenges in numerical simulations, let alone the capturing of the singular free boundary limit. We develop a numerical scheme based on a novel prediction-correction reformulation that can accurately approximate the front propagation even when the nonlinearity is extremely strong. We show that the semi-discrete scheme naturally connects to the free boundary limit equation as $m \to \infty$. With proper spacial discretization, the fully discrete scheme has improved stability, preserves positivity, and implements without nonlinear solvers.

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