Optimal Control Problem with Probabilistic Uncertainty on Initial Positions and Velocities

A. Marigonda[1] **and** **M. Quincampoix**[2]

Abstract: We consider the following optimal control problem

$$\dot{x}(t) = f(x(t), u(t)),\ u(t) \in U,\ t \in [0, T],$$

where $x \in \mathbb{R}^d$ is the state variable, $u(\cdot)$ is the control and $J := g(x(T))$ is a criteria that the controller wishes to minimize.

The main specificities of the optimal control problem we will investigate in the paper lies in the fact that the initial position is not exactly known by the controller but only a probability measure μ_0 is available.

Because of the uncertain initial position at every point of the support of μ_0 corresponds a possible different control. Moreover we allow the "division of mass" i.e. even the initial condition x_0 would be known (namely $\mu_0 = \delta_{x_0}$), it can be splitted in several trajectories by several possible velocities in $f(x_0, U)$ but of course the weight of these trajectories should be remain 1. So the natural state variable of our control problem is probability measure on R^d. The conservation of mass of the trajectory $(\mu_t)_{t\in[0,T]}$ and the controlled dynamics can be summarized in the following dynamical system

$$\begin{cases} \partial_t \mu_t + \operatorname{div}(v_t \mu_t) = 0, & t \in [0, T] \\ \mu|_{t=0} = \mu_0 \\ v_t(x) \in f(x, U) & \text{for } \mu_t \text{ almost every } x \in \mathbb{R}^d \ . \end{cases}$$

The first equation of the above system should be understood in the sense of distribution. So the new state variable is a probability measure, and our aim is to characterize the associate value function through a Hamilton Jacobi Bellman Equation stated on the space probability measures on $\mathbb{R}^d$.

[1] Department of Computer Science,
University of Verona, Strada Le Grazie 15,
37134 Verona, Italy
antonio.marigonda@univr.it

[2] Department of Mathematics,
University of Brest, 6 Avenue Le Gorgeu, 29200 Brest, France
marc.quincampoix@univ-brest.fr