Models of Two Phase Flow in Porous Media

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Abstract: We shall consider models of two phase flow in porous media. A simplified version of the most commonly used model reads

$$\begin{aligned} \partial_t s + \mathrm{div}_{\mathbf{x}}\left(f(s)\mathbf{v}\right) &= 0, \\ \mathrm{div}_{\mathbf{x}}(\mathbf{v}) &= 0, \\ \mathbf{v} &= -\lambda_T(s)\nabla_{\mathbf{x}} p, \end{aligned} \tag{11}$$

where $s \in [0,1]$ denotes the *saturation*, i.e., the fraction of the available pore volume occupied by one of the phases, v denotes the sum velocities of the phases and p denotes the common pressure. The equation (11) must be augmented with suitable initial and boundary conditions. A proof of the existence of weak solutions to (11) has not been found, despite many attempts. Partly because of this, alternative models have been considered, the most obvious is the one obtained by adding a viscous term $\varepsilon\Delta s$ to the right hand side of the first equation. For fixed $\varepsilon > 0$, one then has well-posedness.

Fundamental in deriving the system (11) is Darcy's law, which says that the velocity of a phase is proportional to the pressure of that phase, and that the proportionality coefficient is a function of the saturation. Darcy's law is largely an empirical relation, and its derivation is rather heuristic, especially for multiphase flow. We will consider models resulting from modifications of Darcy's law; concretely the so-called Brinkman model and the Helmholtz regularization, and investigate whether we have well-posedness for these models. We also consider the related issue of convergence of numerical schemes for the resulting equations and their relation to the standard model given by (11).

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