Improving Alternating Direction Augemented Lagrangian Methods for Solving SDP by a Dual Step

M. Santis[1], F. Rendl[2], and A. Wiegele[3]

Abstract: Semidefinite Programs (SDP) can be solved in polynomial time to some fixed prescribed precision, but the computational effort grows both with the number m of constraints and with the order n of the underlying space of symmetric matrices. Interior point methods to solve SDP become impractical both in terms of computation time and memory requirements, once $m \geq 10^4$. Several algorithmic alternatives have been introduced in the literature, several of them based on augmented Lagrangian methods.

Alternating direction augmented Lagrangian (ADAL) methods turned out to be effective for solving SDP, in particular if the number of constraints is large. These methods usually perform a projection onto the cone of semidefinite matrices at each iteration. With the aim of improving the convergence rate of these methods, we propose to update the dual variables before the projection step. The dual SDP problem in standard form is given as

$$\begin{aligned} \max \quad & b^T y \\ \text{s.t.} \quad & C - \mathcal{A}^t y = Z \\ & y \in \mathbb{R}^m,\ Z \succeq 0, \end{aligned}$$

with given data C, b, and $\mathcal{A}$. We consider maximizing the augmented Lagrangian, i.e.,

$$\begin{aligned} \max \quad & L_\sigma(y, Z; X) \\ \text{s.t.} \quad & y \in \mathbb{R}^m,\ Z \succeq 0. \end{aligned}$$

Here, X is fixed, $\sigma > 0$ is the penalty parameter and the augmented Lagrangian is given as $L_\sigma(y, Z; X) := b^T y - \langle Z - C + \mathcal{A}^t y, X\rangle - \frac{\sigma}{2}\|Z - C + \mathcal{A}^t y\|^2$.

Now, instead of considering the constraint $Z \succeq 0$, we use a variable $V \in \mathbb{R}^{n\times k}$ that is a factorization of Z, i.e., $Z = VV^t$, ending up with an unconstrained optimization problem in y and V. Using first-order necessary optimality conditions for this unconstrained problem, we get an explicit formula for y and compute the gradient for the function in V only. Using this information, we compute a search direction to improve the current iterate for Z. This significantly decreases the overall number of iterations within the augmented Lagrangian algorithm, as we will show on instances for the computation of the Lovasz theta number.

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[1] Dipartimento di Ingegneria Informatica Automatica e Gestionale, Sapienza Università di Roma, Via Ariosto, 25, 00185 Roma, Italy
marianna.desantis@uniroma1.it

[2,3] Department of Mathematics, Alpen-Adria-Universität Klagenfurt, Universitätsstr. 65–67, 9020 Klagenfurt, Austria
franz.rendl@aau.at, angelika.wiegele@aau.at