

# Physical Parameter Identification Using Sensors that Combine Different Degrees of Freedom

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**Abstract:** The highly non-linear inverse problem of finding the physical parameters of a dynamical system in terms of its mass, stiffness, and damping matrices from input-output measurements is of both theoretical and practical interests. Applications include detecting damages in bridges or buildings, or detecting tumors in soft tissues. Recent methods take advantage of modern system identification techniques that can identify state-space models of dynamical systems accurately from noisy measurements.

We pioneered an approach that solved the originally nonlinear problem by two successive linear problems. The first linear problem transforms an identified state-space model in some unknown and arbitrary coordinates to physical coordinates, and the second linear problem finds the physical parameters of the structure from the model obtained in first linear problem. Our original solution required a full set of sensors, one per degree of freedom, but recent breakthrough for the two-step approach extended to the case where a full set of sensors is not required. Furthermore, our most recent eigen-decomposition based method offered orders-of-magnitude reduction in computational requirement that enabled the methods to handle systems with very high degrees of freedom.

Up to this point, all available techniques including ours assumed that each measurement (such as displacement, velocity, or acceleration) is associated with an individual degree of freedom. In applications where there is a very large number of sensors, measurements from clusters of sensors that are in proximity to each other might be combined together. Other scenarios include the use of spatially distributed sensors such as strain gages that combine information from more than one degree of freedom. It turns out that this slight modification to the mathematical problem definition requires a non-trivial extension to the available solutions. This paper provides such an extension to our most recently developed methods.

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