On Regularity and Discretization of Affine Optimal Control Problems

V. M. Veliov[1]

Abstract: The first part of the talk is devoted to some new results related to Lipschitz (or Hölder) stability of the solutions of optimal control problems which are affine with respect to the control. Although for "coercive" problems the issue is well understood, in the affine case it is still under investigation; we mention contributions by U. Felgenhauer, L. Poggiolini, and G. Stefani during the years 2003–2015, concerning the property of "structural stability". We investigate related but different properties, namely *strong bi-metric regularity* and *strong Hölder sub-regularity*. We begin with a motivation and explanation for the general definition of strong bi-metric regularity, and a Lyusternik-Graves type theorem for this property. We interpret these results in the case of affine optimal control problems by choosing appropriate space settings. In particular, the property of strong bi-metric regularity turns out to be equivalent to the same property of the linearized problem around a reference solution (the proof is much more involved than that for "coercive" problems). Then we give sufficient conditions for bi-metric regularity of affine linear-quadratic problems, as well as sufficient conditions for (Lipschitz or Hölder) strong sub-regularity.

In the second part of the talk we present or indicate some applications to numerical methods for affine problems. We focus on two issues: (i) a Newton type method for which convergence analysis is performed; (ii) discretization scheme with higher order accuracy than the Euler scheme.

Both issues are well studied for problems satisfying strong coercivity conditions, but if the optimal control is of bang-bang type the convergence of the Newton method have not been studied, and the known time-discretization methods ensure at most first order accuracy (with respect to the discretization step).

The talk is based on joint works with J. Preininger and T. Scarinci.

[1] ORCOS, Institute of Statistics and Mathematical Methods in Economics
Vienna University of Technology
Wiedner Hauptstr. 8-10, 1040 Vienna, Austria
vladimir.veliov@tuwien.ac.at