The Threshold for the Existence of a Global Holderian Error Bound of a Polynomial Function

H. H. Vui[1]

Abstract: Let $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ be a polynomial. For $t \in \mathbb{R}$, let $M_f(t) := \{x \in \mathbb{R}^n : f(x) \leq t\}$.

We say that $M_f(t)$ has a global holderian error bound (GHEB for short) if there exist $\alpha, \beta, c > o$ such that

$$(f(x) - t)_+^{\alpha} + (f(x) - t)_+^{\beta} \geq c\mathrm{dist}(\mathrm{x}, \mathrm{M_f(t)}) \tag{15}$$

for all $x \in \mathbb{R}^n$, where $(f(x) - t)_+ = \max\{f(x) - t, 0\}$.

We define the *threshold* for the existence of a global holderian error bound of f, denoted by $S(f)$, as follows:

- $S(f) = -\infty$, if $M_f(t)$ has a GHEB for every $t \in \mathbb{R}$;
- $S(f) = +\infty$, if $M_f(t)$ does not have a GHEB for any $t \in \mathbb{R}$;
- $S(f) = \inf\{t : M_f(t) \text{ has a GHEB}\}$ if $S(f) \neq \pm\infty$.

In this talk we give sufficient conditions for $S(f) = -\infty$ or $S(f) \in \mathbb{R}$. It follows from these conditions that if f is a polynomial in two variables, then either $S(f) = -\infty$ or $S(f)$ is finite. Next, using the Newton-Puiseux expansions at infinity of algebraic curves, we give a method of computing the exact value of $S(f)$ of any polynomial f in two variables. This method also computes explicitly, in the case of two variables, the best possible exponents α and β in Inequality (15).

[1] Institute of Mathematics and Applied Science (TIMAS)
Thang Long University
Nghiem Xuan Yem Road, Hanoi, Vietnam
hhvui@math.ac.vn